What can children learn by choosing their favorite kinds of apples? Discover some of the many ways that graphing can lead to better math understandings as well as many other important thinking skills.

Young Children Use Graphs to Build Mathematical Reasoning

Mark J. Larson and David J. Whitin

Mathematical, scientific, and technological knowledge is critical for people in a 21st Century world that is dependent upon a global interconnectedness and a knowledge-based economy (National Science Board, 2007). This is the kind of knowledge that will power innovations and drive decision making in the years ahead. Schools are therefore being called upon to devise a mathematics curriculum that is inspiring, rigorous, and thoughtfully articulated so that more students will be successful in math and pursue careers in science, technology, engineering, and mathematics (STEM [Science, Technology, Engineering, and Mathematics] Education Coalition, 2009).

Gathering, collecting, and interpreting numerical information is a necessary skill.

One curriculum area that is receiving increased attention in this national STEM initiative is the topic of graphing (National Council of Teachers of Mathematics, 2006; National Science Board, 2007). Gathering, collecting, and interpreting numerical information is deemed an ever-increasingly necessary skill in a world that is inundated by data.

Graphing is a topic that is also clearly connected to the joint position statement on early childhood mathematics developed by the National Association for the Education of Young Children and the National Council of Teachers of Mathematics (NAEYC, 2002). This document recommends high-quality mathematics education content for 3- to 6-year-old children. Within these recommendations are three important reasons to include graphing in early childhood classrooms.

- Teachers can *enhance children's interest in mathematics* by creating graphs about familiar routines and classroom events.
- Teachers can *emphasize the pervasiveness of graphs and mathematics* in the world by integrating their use across subject fields such as science, visual arts, technology, engineering, social studies, and language arts.
- Graphing provides a meaningful opportunity for children to *represent and communicate important mathematical relationships*. Some of these relationships include equality, inequality (more/less), and the associative property in addition. This property, also referred to as a "grouping property," states that a change in order in the grouping of three or more addends does not change the sum: $a + (b + c) = (a + b) + c$.

This article describes how one teacher, who will be called Mrs. Elias, brought a variety of mathematical benefits to life with her kindergarten students as they created and interpreted a graph about their favorite apples. Her story also highlights the key role that teachers play in posing open-ended questions that enable children to make their mathematical thinking visible (Berk & Winsler, 1995; Copple & Bredekamp, 2009).

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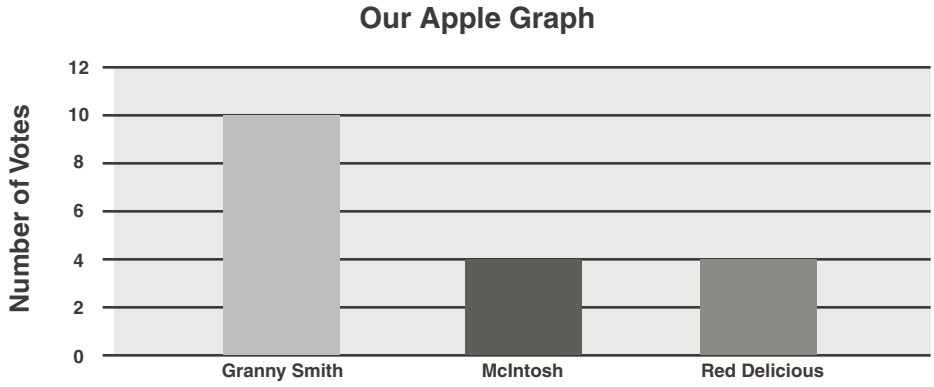
Kindergarteners Taste, See, and Think

Late in the fall, Mrs. Elias brought apples to class and invited the children to taste slices of three varieties: Red Delicious, McIntosh, and Granny Smith. After the children had tasted the apples, they colored and cut out an apple—either red (Red Delicious), green (Granny Smith), or red and green (McIntosh)—to represent their favorite type of apple. Students then placed their apple cut-outs on a simple chart that listed the three choices, much like they had done before with other topics.

The next day, Mrs. Elias worked with the children to represent their data in the form of a bar graph (see Figure 1). First, they counted the number of apples in each section of the chart. As Mrs. Elias drew the graph, she explained what the numbers on the left side represented. She deliberately used increments of two because the children were beginning to do skip counting by twos. The children suggested using the same colors as their apples to make the three bars. They used rulers to measure how long to cut the colored paper strips to go on the graph. Children carefully glued the strips in place.

Figure 1. Graph of favorite apples.



After the graph was complete, Mrs. Elias invited the children to interpret their data. She asked, "What do you notice about our graph?" Some of the responses are listed here.

- Geena (using a counting stick): "One, two, three, four, five, six, seven, eight, nine, ten. There's 10 green apples."
- Michael: "The green's the tallest."
- Stephen: "The green has a lot (counts them by 2s)—there are 10."
- Liam: "Um. The Red Delicious and the McIntosh are the same—they have four."
- Sabrina: "There's four McIntosh and four Red Delicious."

Mrs. Elias's use of an open-ended question, "What do you notice about our graph?" enabled the children to draw their own interpretations about the mathematical relationships. Sometimes, teachers do the cognitive work for children by determining the relationships ahead of time, so they might ask: "How many more people voted for Granny Smith than McIntosh?" This comparison could certainly be made later on. However, initially children who are encouraged to talk about what they notice have the opportunity to construct their own mathematical relationships. In this conversation:

- Geena used a counting stick to verify the total number for green.
- Michael and Stephen described the green total in terms of size ("tallest") and number ("a lot").
- Stephen employed skip counting to get to the total of 10.
- Liam and Sabrina demonstrated their understanding of the concept of equivalence because they described the tallies for Red Delicious and McIntosh as being "the same" because they both "have four."

Nancy P. Alexander

Schools are being called upon to devise a mathematics curriculum that is inspiring, rigorous, and thoughtfully articulated so that more students will be successful in math and pursue careers in science, technology, engineering, and mathematics.

Equivalence is one of the most important concepts for children to learn in elementary school (NCTM, 2000), and graphing can provide a meaningful context for young children to use this concept for a real purpose (Dacey & Eston, 1999). In this case the children compared two bars on the graph to find equivalence. In other graphs with different numbers the children might combine other bars to make equivalence, such as adding of bars 5, 3, and 2 to match a 10 bar. They might also subtract 2 from the 10 bar to make it equivalent to a combination of a 5 bar and a 3 bar. Children enjoy the challenge of "making the same number" by combining quantities in different ways.

Next Mathew used the counter to add up all the apples and announced the total of 18. When Mrs. Elias asked, "Why do you suppose there are 18 altogether?" he replied, "Because there's 18 children in our class!" Mathew was using his understanding of one-to-one correspondence to answer the teacher's question about equivalence.

As the discussion continued, Mrs. Elias challenged the children to describe a general comparison they made in more detail. When Marissa at first observed, "The Granny Smith is more than Red Delicious or the McIntosh," the teacher extended this idea by asking, "I wonder *how many more* Granny Smith apples there are than the Red Delicious or the McIntosh? Who can show us?" Although determining exactly how many more can be a difficult task for children at this age, the teacher wanted to assess their abilities to explain this relationship.

Marissa volunteered to share her reasoning and began by stating that there are four McIntosh and four Red Delicious and that "there are a lot of greens. See?" After she had directed everyone's attention to this general comparison, she used the pointer to make a finer distinction. She moved the pointer across the bottom rows of all three columns, emphasizing this initial equivalence of 1. She then compared all second rows, third rows, and fourth rows, using the matching strategy to show that at this point all the columns were equivalent.

Make mathematical thinking visible.

The teacher then asked, "Can you tell us how many extra greens there are?" Marissa counted the green apples above the pointer and responded, "Five."

The teacher then said, "Check carefully."

Marissa recounted and said, "Oops, it's six!"

During this part of the conversation Mrs. Elias built on Marissa's initial observation by challenging the children to determine exactly "how many more?" She also prompted Marissa to check her counting again, giving her the opportunity to fix her answer. Children develop positive attitudes about themselves as problem solvers when they are given the chance to revisit and revise their thinking on their own.

Children Show Data in Many Ways

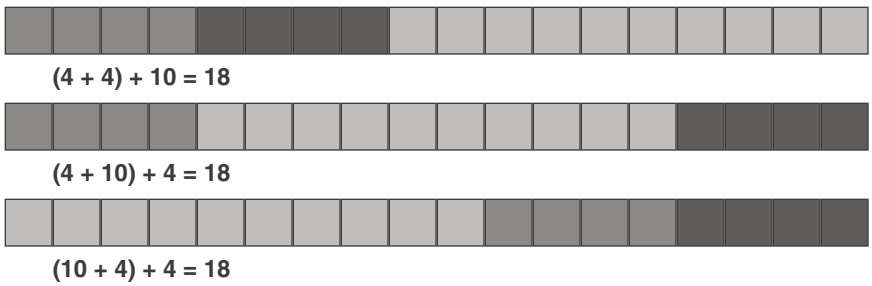
The next day, Mrs. Elias invited the children to represent the same data in a different way with Unifix® cubes. She knew that the way a set of data is displayed can often influence the way the results are interpreted and talked about (Whitin & Whitin, 2003). In this case, the Unifix cubes led children to use different mathematical language and were a tool for concretely using the associative property in addition.

First, the children snapped together 4 red cubes to represent Red Delicious, 4 maroon cubes to represent McIntosh, and 10 green cubes to represent Granny Smith apples. The stacks were stood up on a table for all to see. When Mrs. Elias asked the children to describe what they saw, they used some different language than they had the day before.

- Geraldo: "Green is so much higher."
- Patrick: "McIntosh and the red are even."
- Shanique: "If you put two 4's together it makes 8."

What led to this new language? One tower did look "higher" and the other two towers appeared "even." The children also capitalized on a unique feature of these cubes by snapping together the two 4s to make 8, a relationship that was not mentioned

Figure 2. Children rearranged three addends and discovered that the sum is always the same (the associative property).



Subjects & Predicates

Ask open-ended questions. Children who are encouraged to talk about what they notice have the opportunity to construct their own mathematical relationships.

during the discussion of the bar graph. After these two 4s were combined, other children began putting together the three stacks in different ways (Figure 2).

In this informal way, the children were demonstrating their understanding of the associative property of addition (rearranging the addends does not change the sum). Mrs. Elias built on the children's interest by continuing to encourage multiple solutions. As they represented data in more than one way, children used different language and noted other mathematical relationships.

This teacher then challenged children to think about the limitations of their graph. She knew that a healthy skepticism about numerical information was an important disposition for children to have when interpreting data (Whitin, 2006). She asked them, "What does the graph **not** show us? What information is **not** told by our graph?"

Thought-provoking questions like these encourage children to consider the limitations of a set of data. Every graph is only a partial representation about a topic or issue (Best, 2004). Realizing the parameters of a graph—what it represents and does *not* represent—gives children a critical perspective on what conclusions can and cannot be drawn. These children shared some important insights about the limitations of their graph.

- Ashley: "There's no yellow ones."
- Marissa: "It doesn't tell us favorite toys."
- Conner: "It doesn't tell us if we tried the same apples."
- Georgiana: "We didn't write our names on the apples."

Ashley noted that only 3 colors of apples were sampled so that there was no data on yellow apples. Georgiana realized that a potential layer of information was lacking, e.g. the names of the child who selected each apple. Conner claimed that each child might not have tasted from the same apple, insinuating that perhaps there was a slight difference in taste among the same kind of apple. And Marissa asserted that the graph was not telling them about other interests of children, such as favorite toys.

Healthy skepticism about numerical information is important.

These comments help to demonstrate to children that in any given set of data there is always something missing. Certain relationships are revealed while others are omitted (Curcio, 2001). When graphs are tied to a familiar context, such as apples, children are better able to uncover these limitations themselves.

Mrs. Elias then expanded the discussion by asking, "We know what our favorite tasting apple is. Who has another question you could ask your classmates?"

- Tommy: "If they like apple juice."
- Marissa: "If their tooth fell out."
- Patrick: "If someone likes blue or red. Cause I like blue."
- Georgiana: "What number they like. I like 10."
- Geena: "What the first letter of their name is."
- Conner: "I would like to know if they got chicken pox or not."
- Liam: "About centers. What center they like best. I like blocks."
- James: "I would ask them if they had a pet."
- Shelly: "What their favorite animal is."

By challenging the children to extend their thinking about surveys, Mrs. Elias was drawing out their original interests and ideas. In doing so, children were more likely to feel a sense of ownership that would lead them to carry out research on these new topics for graphing. Clearly the children were curious about their classmates' opinions and interested in finding out more about each other's ideas. They were becoming important collectors of information.

Key Teaching Strategies With Graphs

Today's world is awash with numbers. People use numbers continuously to frame arguments, justify policies, and attempt to change others' opinions. For this reason, it is essential that children have regular and rich opportunities to gather, represent, and interpret their own data. These experiences can be integrated across the curriculum (National Science Board, 2007) so that children use data for a range of authentic purposes in many different contexts.

The observational case study with young children that has been described here highlights several key instructional strategies that teachers can use to promote this important mathematical thinking.

Connect mathematics to regular classroom events to make learning both meaningful and memorable. Concepts and strategies arise naturally in authentic contexts, including children's literature (see Table 1). Teachers can tie graphing experiences to classroom experiences and routines with survey explorations such as these:

- What would you like to serve for snack next week?
- Which character would you like to be in (shared story)?
- How many people have had a chance to feed our fish? How many are waiting for a turn?
- What are some ways we could record each day's weather (cloud cover, temperature, wind speed)?

Graphing experiences can also be connected to children's interests and experiences, including questions like these:

- What activities do you do after school?
- Which is your favorite piece of playground equipment?
- Who has lost any teeth? How many? When?
- What kinds of animals are your pets?

Use graphs to help make class decisions, which also impart important lessons about democracy, responsibility, and respecting the perspectives of others.

- Which of these three books would you like to read aloud first?
- What kind of sandwich could we prepare for our class party?
- Which of the names you suggested shall we choose for our guinea pig?

During their interpretations of the apple graphs, children used the concept of equivalence as well as the strategy of comparing as they noted the relationships between three columns. They also undoubtedly realized that other children have preferences that may differ from their own, and that their comments and solutions are respected by each other.

Pose open-ended questions to elicit a variety of types of responses. Questions provide multiple points of entry for children with varied abilities

Table 1.
Math-related children's literature that can be a catalyst for graphs.

- Baer, E. (1992). *This is the way we go to school*. NY: Scholastic. (How do you travel to school?)
- Baer, E. (1995). *This is the way we eat our lunch*. NY: Scholastic. (What do you prefer for lunch?)
- Freymann, S. (2006). *Fast food*. NY: Arthur A. Levine. (What is your favorite fruit? Vegetable?)
- Grejneic, M. (1995). *What do you like?* NY: North South Books. (To what kinds of songs do you listen?)
- Henkes, K. (1996). *Chrysanthemum*. NY: Mulberry. (How many letters are in your name?)
- Morris, A. (1999). *Teamwork*. NY: William Morris. (How do you help your family?)
- Morris, A. (2000). *Families*. NY: HarperCollins. (What people are in your family?)
- Stinson, K. (1988). *Red is best*. NY: Firefly Books. (What is your favorite color?)

and prior experiences. For instance, the question, "What do you notice?" invites all children to enter the conversation by sharing their initial observations. Some may focus on the colors. Others will count the total for each bar, while others may compare bars, or recombine parts of bars for further comparison.

Open-ended questions, and the variety of responses, enable children to recognize a range of mathematical relationships in a graph, to elaborate on their ideas, and can help teachers better assess what children know and understand so they can plan learning experiences that further expand children's horizons.

Invite children to represent the same data in at least two different ways so they have the opportunity to highlight different numerical relationships. When the children used Unifix® cubes to represent their data, they began combining the two 4s to make 8, as well as to rearrange the three stacks in different ways to demonstrate their understanding of the associative property. Other ways they could represent the data include preparing pie charts, sculpting apples with modeling compound, or writing stories about their apple choices.

Encourage children to be aware of the limitations of their graphs to build a healthy skepticism about numerical information. By asking children "What does your graph **not** say?" teachers instill in children a critical eye and bolster their mathematical reasoning. Other questions teachers might ask include:

- What else can we conclude from these findings?
- What other information could we add to make sure everyone's opinion is represented? (more kinds of apples, a choice of no apples)
- What are other ways to interpret the data? (For example, almost half of the children liked red apples and half liked green apples, or that about half of the children liked tart apples and half preferred sweeter ones.)

Subjects & Predicates

Thought-provoking questions encourage children to consider the limitations of data. Every graph is only a partial representation about a topic or issue.

Instructional strategies that include graphs help to build the kind of robust, challenging, and relevant mathematics learning community that supports students to be confident and competent mathematical thinkers.

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Is there a particular topic that you would like to see discussed? Send your letter to info@southernearlychildhood.org, or by mail to PO Box 55930, Little Rock, AR 72215-5930, and let us know what's on your mind. If your letter is selected for one of our monthly discussions, we'll contact you prior to posting your letter.

Students and SECA

In order to support future early childhood professionals, we have launched a Facebook page just for our student members. This new page will serve to provide students with the mentoring and guidance they will need as they enter the early childhood field. The key feature of this new page will be the ***live*** monthly mentored discussions, during which an experienced early childhood professional will open up a topic for discussion and be available to answer questions and discuss any comments —in real time—that students may have. Each month, we'll send our student members an email giving the date, time, and topic of the upcoming live discussion. Students can continue the discussion with each other after the live discussion has ended, and the mentor will periodically check the page to address any questions or comments that have been posted.

If you're a student member of SECA, look for an email about the first discussion to arrive in your inbox soon. For those experienced professionals interested in serving as mentors, send an email to info@southernearlychildhood.org. We think this is a truly unique way for us to reach out to and support our students, and we hope you'll join in!

Put These Ideas Into Practice!

Young Children Use Graphs to Build Mathematical Reasoning

Mark J. Larson and David J. Whitin

Key Teaching Strategies With Graphs

Connect mathematics to regular classroom events

- Which character would you like to be in (shared story)?
- How many people have fed our fish? How many are waiting for a turn?
- What activities do you do after school?
- Who has lost any teeth? How many? When?
- What kind of sandwich could we prepare for our class party?
- Which of the names you suggested shall we choose for our guinea pig?

Pose open-ended questions about graphs, such as, "What do you notice?"

Invite children to represent the same data in at least two different ways, such as on graphs as well as with blocks and in their art.

Encourage children to be aware of the limitations of their graphs. Ask children "What does your graph not say?"

Integrate graphing into science learning

Science is a particularly relevant content area for using graphs. Graphing can often be connected to measuring and classifying, such as these examples.

- How many objects floated? How many sank?
- How much does our guinea pig weigh each month?
- What is the outside temperature each day?
- What is your favorite fruit? Vegetable?
- How tall is your radish plant? How has it changed?
- How many red leaves did you find outside? How many orange? How many yellow?
- How many of these plastic dinosaurs are carnivores? Herbivores? Omnivores?
- How many of these rocks are smooth? How many are rough?

Promote professional development about graphing

Directors can encourage teachers to analyze graphs and to closely examine children's work. In-service professional development opportunities could be planned around experiences like these.

- Ask teachers to bring graphs from newspapers or on the Internet. Make copies of some of them. In small groups, teachers read and interpret them.
- Discuss graphs using open-ended questions such as these:
  - What do you notice about this graph?
  - What do you find interesting?
  - What does the graph NOT say?
- Invite teachers to bring examples of some children's graphs as well as a transcript (or a recording) of a recent graphing conversation. What do teachers notice about the children's observations and interpretations? How could children's learning be further extended?
- Ask teachers to evaluate children's work by recording three pluses and a wish: What are three things you appreciate about this child's work? What is a next step for you as a teacher to support this child's learning?